

# MIMO-OFDM Channel Estimation with Minimum Differential Feedback for Time-Correlated Rayleigh Block-Fading Channels

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**Abstract:** In Multi-Input Multi-Output (MIMO) based cognitive radio (CR) systems, with the increasing demand for data rate and reliability in Wireless communications and devices, several issues become very important like bandwidth efficiency, quality of service and radio coverage. We first derive the closed-form expression of the minimum differential feedback rate to achieve the maximum ergodic capacity in the presence of channel estimation errors and quantization distortion at the receiver. With the feedback-channel transmission rate constraint, in the periodic feedback system, we further investigate the relationship of the ergodic capacity and the differential feedback interval. First formulate the pilot design as a new optimization problem. Instead of minimizing the mean-square error (MSE) of the least-squares (LS) channel estimator, we minimize an upper bound which is related to this MSE. We then propose an efficient scheme to solve the optimization problem. This reliability is in the context of the channel estimation in our case. With the MIMO concept we improve the bitrate and BER of the overall system. Simulation results show that the pilot index sequences obtained by the proposed method exhibit significantly better performance than those obtained by existing pilot design methods.

**Keywords—** Channel state information, MIMO, pilot design, channel estimation, Rayleigh block-fading channels.

## I. INTRODUCTION

Channel state information (CSI) feedback from the receiver to the transmitter has been intensively studied with great interest due to its potential benefits to the multiple input multiple output (MIMO) system. CSI can be utilized by a variety of channel adaptive techniques (e.g., water-filling, beamforming, precoding, etc.) at the transmitter to enhance the spectral efficiency as well as the robustness of the system, especially, in the frequency division duplexing (FDD) mode. As the transmission rate of the feedback channel is normally very limited, the infinite feedback of CSI is hard to realize in practice.

Therefore, it is important to investigate how to decrease the amount of feedback signalling overhead to meet the uplink feedback channel requirements. As a result, CSI feedback reduction has attracted lots of attention in recent years [1], [2]. Specifically, when the wireless channel experiences time-correlated fading [3], typically represented by a Markov random process [4], the amount of CSI feedback can be largely reduced. In [5], a number of feedback reduction schemes were summarized, considering the lossy compression scheme exploiting the properties of fading process as the best choice. The use of the radio spectrum is regulated by governmental rules. Almost all parts of the radio spectrum are licensed today. The FCC published a thesis in [6] where the spectrum use in the United States is presented in the aim of better spectrum utilization. On the other hand, studies have shown that major licensed bands are largely underutilized. This has initiated the idea of cognitive radio (CR), where secondary users are allowed to utilize the licensed bands without causing harmful interference to the licensed or primary users [7]. The CR technology has the potential to significantly increase the efficiency of the spectrum utilization while maintaining the QoS requirement of the primary users. Multi-Input Multi-Output (MIMO) is the most widely used technology in current and future wireless communication systems. Since OFDM can naturally provide flexibility to fill in spectrum holes over a wide bandwidth, it has been considered as one of the best candidates for the physical layer of CR systems [8], [9]. The channel state information (CSI) is crucial for CR systems [10].

In practice, the CSI can be estimated by using pilot tones. The selection of pilot tones will significantly affect the channel estimation performance. For conventional OFDM systems where all subcarriers can be used for transmission, the issue of pilot design

has been well studied [11]. However, these methods are not effective for OFDM-based CR systems, since the subcarriers used by the primary users cannot be employed by the secondary users. Hence the available subcarriers for the secondary users may be non-contiguous which brings new challenges to pilot design.

The simplest pilot design for OFDM-based CR system is to pre-design pilot tones for the conventional OFDM system and then deactivate those tones already used by the primary users according to the result of spectrum sensing. At the receiver, the remaining activated pilot tones are used for channel estimation. Directly implementing channel estimation based on such pilot pattern leads to poor performance [12]. To obtain satisfactory channel estimation performance, a shift pilot scheme is proposed in [13]. After pre-designing pilot tones and deactivating some of them according to the spectrum sensing result, this scheme selects some activated data subcarriers as the new pilot tones. Specifically, when a pilot tone is deactivated, its nearest activated data subcarrier is then used as the new pilot tone. The similar idea can be also found in [14], [15]. Although such a scheme outperforms the aforementioned one, it cannot provide satisfactory performance in all cases. This is mainly because the positions of pilot tones are not optimized.

Instead of minimizing the mean-square error (MSE) of the least-squares (LS) channel estimator itself, we minimize an upper bound which is related to this MSE. The MSE and BER are used as measure in judging the performance of each pilot pattern. According to that performance of pilot pattern has been implemented in MIMO based CR system. Virtual pilot concept has been reviewed and implemented in all proposed pilot patterns. The advantages and disadvantages of virtual pilot concept has been discussed through our simulation results analysis. The decision of the best performing pattern is made by observing the MSE and BER obtained from the simulation results in the case where no LU is active. The hexagonal pattern shows the lowest MSE and BER compared to the other two patterns. Hence the hexagonal pattern is chosen to be implemented in the CR system. The remainder of this paper is organized as follows. In Section II, we briefly introduce the system model. In Section III, we describe the LS channel estimator. The proposed new pilot design method is described in Section IV. Section V presents simulation results and finally, conclusions are drawn in Section VI.

## II. SYSTEM MODEL

### A. MIMO model with CR system

The MIMO-based CR system under consideration is shown in Fig.1. The pilots are designed according to

the result of the spectrum sensing. After subcarrier assignment where the subcarriers occupied by the primary users are deactivated, pilot symbols are inserted and the data are modulated on the remaining activated subcarriers. We employ the MIMO concept in our simulation platform because it has been proven that MIMO can achieve a major breakthrough in providing reliable wireless communication links. This reliability is in the context of the channel estimation in our case. With the MIMO concept we improve the bitrate and BER of the overall system. MIMO is capable of this improvement, because of the property of multiple transmission multiple reception. This property is a form of spatial diversity. This diversity is the most effective technique to accomplish reliable communication over the wireless channel and combating with fading, because it provides the receiver with multiple copies of the transmitted signal. Those multiple copies are independently faded. If at least one copy of the transmitted signal is received correctly, we will have the transmitted signal back. This property improves the BER significantly (low BER) as shown in chapter 6. Beside this, MIMO increases the channel capacity also, which means more throughputs. There are different ways to exploit multiple antennas at both sides of the communication channel. To improve the transmission reliability, the transmit antennas should be used such that transmit diversity is achieved. The transmission rate is comparable to the one obtained in SISO. To improve the transmission rate, independent signals are transmitted from the different transmit antennas. i.e. there is no correlation between the transmitted signals from the different antennas. In this case the reliability is not much improved [10].

$$y_r(t) = \sum_{k=0}^{K-1} \sum_{i=1}^M h_{i,r}^{(k)}(t) s_i(t-k) + n_r(t) \quad (1)$$

Where  $s_i(t)$  is the transmitted signal from antenna  $i$  at time  $t$ ,  $h_{i,r}^{(k)}(t)$  is the channel coefficient for the  $k$ th path from transmit antenna  $i$  to receive antenna  $r$  at time  $t$ .  $n_r(t)$  is the Additive White Gaussian Noise. For mobile communications, the channel tap coefficients are random variables. In case the wireless channel varies very slowly, the tap coefficients remain constant for each frame of data. For Rayleigh fading channels, the channel tap coefficients are modeled as complex Gaussian random variables which have zero mean. The different channel taps are assumed to be independent. The average channel gains for different paths are determined from the power Delay profile of the wireless channel. In this work we assume that the channel tap powers decays exponentially. Hence we use the exponential power delay profile. If the MIMO-OFDM system has  $N_c$  subcarriers and the fading coefficients are spatially uncorrelated and that

the fading coefficients remain constant during one OFDM symbol. Then the transmitted signal over M antennas can be represented by a matrix XOFDM with dimensions  $N_c \times M$ . A symbol transmitted at subcarrier n on transmit antenna is  $x_i(n)$ .

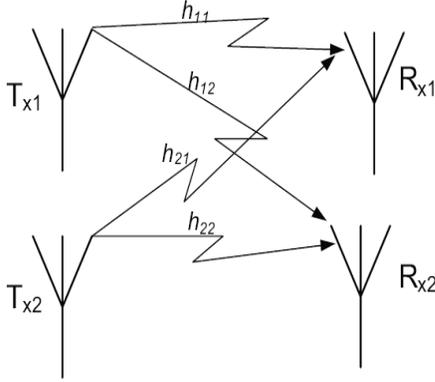


Figure 1 : MIMO setup

At the receiver after applying FFT and removing the cyclic prefix, the resulting signal at the jth receive antenna for the nth subcarrier will be:

$$y_r(n) = \sum_{i=1}^M x_i(n) H_{i,r}(n) + n_r(n) \quad (2)$$

where  $H_{i,r}(n)$  is the channel coefficient from the ith transmit to the rth receive antenna for the nth subcarrier and is given by:

$$H_{i,r}(n) = \sum_{k=0}^{K-1} h_{i,r}^{(k)} e^{-j2\pi nk/N_c} \quad (3)$$

where we can denote the channel coefficients for the (i, r)th links by:

$$H_{i,r} = [H_{i,r}(0) H_{i,r}(1) \dots H_{i,r}(N_c - 1)] \quad (4)$$

The vector of K independent fading coefficients of the different taps can be represented by:

$$A_{i,r} = [h_{i,r}^{(0)} h_{i,r}^{(1)} \dots h_{i,r}^{(K-1)}] \quad (5)$$

Hence we can see that the equivalent channel coefficients are:

$$H_{i,r} = W A_{i,r} \quad (6)$$

Where W is the FFT matrix. In (6) the fading coefficients for distinct subcarriers are different but dependent. The maximum rank of the (frequency domain) channel matrix is equal to the number of taps K, and is usually low. The time domain signals at the receive antennas can be represented as:

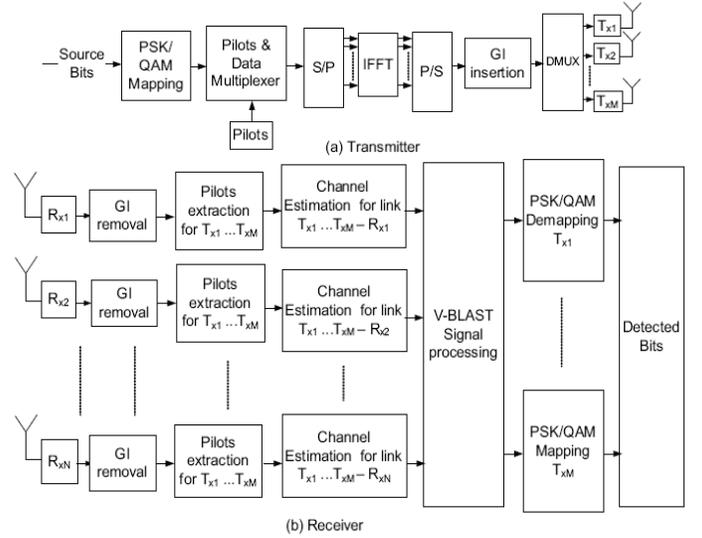


Figure 2 : MIMO System model (a) Transmitter and (b) Receiver

$$y_1 = s_1 * CIR_{11} + s_2 * CIR_{21} + n_1 \quad (7)$$

$$y_2 = s_1 * CIR_{12} + s_2 * CIR_{22} + n_2$$

Where  $y_1$  and  $y_2$  are the vectors received by receive antennas  $R_{X1}$  and  $R_{X2}$  respectively. The transmitted signal vectors by transmit antennas  $T_{X1}$  and  $T_{X2}$  are represented by  $s_1$  and  $s_2$  where all variables are in time domain.  $CIR_{11}$ ,  $CIR_{12}$ ,  $CIR_{21}$  and  $CIR_{22}$  are the channel impulse responses of the different sub-channels, while  $n_1$  and  $n_2$  are the noise vectors (AWGN) with zero mean on the two receive antennas. Those two noise vectors are independent of each other. Putting (3.28) in matrix form gives:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} CIR_{11} & CIR_{12} \\ CIR_{21} & CIR_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad (8)$$

Taking the FFT of the CIR gives the frequency domain channel matrix CTF as:

$$CTF_{2 \times 2} = \begin{pmatrix} CTF_{11} & CTF_{21} \\ CTF_{12} & CTF_{22} \end{pmatrix} \quad (9)$$

## B. The proposed pilot design method

### C. Extension to MIMO-OFDM Systems

The proposed pilot design scheme can be readily extended to MIMO systems. In this subsection, we present two extension methods. Let  $\mathcal{P}^q = \{p^q_0, \dots, p^q_{P-1}\}$  denote the pilot tone placement at the q-th transmit antenna and assume that  $\mathcal{P}^q \cap \mathcal{P}^r = \emptyset$  for  $q \neq r$ , i.e., pilot tone placements for different transmit antennas are orthogonal. The direct extension of the proposed scheme is to design  $\{\mathcal{P}^q\}_{q=1}^N$  sequentially, where  $N_t$  is the number of transmit antennas. The following Extension Method 1 describes the whole procedure.

1) Initialize  $\mathcal{N}^1 = \mathcal{N}$ . Set  $q = 1$ .  
 2) Obtain  $\mathcal{P}^q$  by using Algorithm 2, where  $\mathcal{N}$  and  $G_n$  are replaced by  $\mathcal{N}^q$  and  $G_n^q = \sum_{m=0}^{n-1} g(P_m^q - P_n^q)$ , respectively.  
 3) Let  $\mathcal{N}^{q+1} = \mathcal{N} \setminus \cup_{i=1}^q \mathcal{P}^i$ . Set  $q = q+1$ . If  $q = N_t+1$ , stop, otherwise, return to step 2).  
 Note that  $\mathcal{N}^{N_t} \subset \dots \subset \mathcal{N}^1$ . The above method may lead to “unfairness” among  $N_t$  transmit antennas. For example, for the first transmit antenna, pilot tones can be chosen from  $|\mathcal{N}^1| = N$  possible subcarriers, while for the last transmit antenna, pilot tones can only be chosen from  $|\mathcal{N}^{N_t}| = N - (N_t - 1)$  possible subcarriers. In some cases, the last designed pilot placement may cause much worse performance than the first designed one. Since the system performance is an average performance, reducing the performance gap between the best and the worst may improve the final performance. Consequently, we propose Extension Method 2 that can enhance the fairness among  $N_t$  transmit antennas. In Extension Method 2, pilot tones of different transmit antennas are designed in turn. Specifically, we first design  $\{p_0^q, p_1^q\}_{q=1}^{N_t}$  (i.e., the first two pilot tones of all transmit antennas) in the order of  $\{p_0^1, p_1^1\}, \dots, \{p_0^{N_t}, p_1^{N_t}\}$ . Then, we design  $\{p_2^q\}_{q=1}^{N_t}$  in the order of  $p_2^{N_t}, p_2^{N_t-1}, \dots, p_2^1$ .

After that, we design  $\{p_3^q\}_{q=1}^{N_t}$  in the order of  $p_3^1, p_3^2, \dots, p_3^{N_t}$  and so on. In other words, for the  $q$ -th pilot tone, if the design order is  $p_1^1, p_1^2, \dots, p_1^{N_t}$ , then for the  $(q+1)$ -th pilot tone, the design order will be  $p_{q+1}^{N_t}, p_{q+1}^{N_t-1}, \dots, p_{q+1}^1$ . The whole procedure is summarized as follows.

- 1) Initialize  $\mathcal{N}^1 = \mathcal{N}$ . Set  $q = 1$  and  $n = 1$ .
- 2) Obtain  $p_0^q$  and  $p_1^q$  by solving (15), where  $\mathcal{N}$  and  $G_1$  are replaced by  $\mathcal{N}^q$  and  $G_1^q$ , respectively. Initialize  $\{p_0^q, p_1^q\}$ .
- 3) Set  $\mathcal{N}^{q+1} = \mathcal{N}^q \setminus \mathcal{P}^q$  and  $q = q + 1$ . If  $q = N_t + 1$ , let  $\mathcal{N}^{N_t} = \mathcal{N}^{N_t+1}$  and  $q = N_t$ , and go to step 4), otherwise, return to step 2).
- 4) Set  $n = n+1$ . If  $n = P$ , stop, otherwise, go to step 5).
- 5) Obtain  $p_n^q$  by solving (16), where  $G_n$  is replaced by  $G_n^q$  and  $\mathcal{N} \setminus \mathcal{P}$  is replaced by  $\mathcal{N}^q$ . Update  $P^q = P^q \cup \{p_n^q\}$ .
- 6) If  $n$  is even, set  $\mathcal{N}^{q-1} = \mathcal{N}^q \setminus \mathcal{P}^q$  and  $q = q - 1$ . If  $q = 0$ , let  $\mathcal{N}^1 = \mathcal{N}^0$  and  $q = 1$ , and return to step 4).

- 7) If  $n$  is odd, set  $\mathcal{N}^{q+1} = \mathcal{N}^q \setminus \mathcal{P}^q$  and  $q = q + 1$ . If  $q = N_t + 1$ , let  $\mathcal{N}^{N_t} = \mathcal{N}^{N_t+1}$  and  $q = N_t$ , and return to step 4).
- 8) Return to step 5).

### III. SIMULATION RESULTS

Figures 3 and 4 show the MSE and BER performance of the above five methods, respectively. The LS channel estimator described in Section III is applied. From the figures it can be seen that Method A has the poorest performance and it is virtually unusable for the MIMO-based CR system.

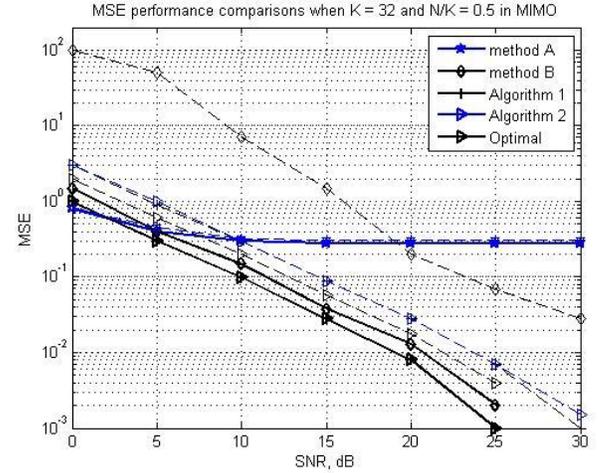


Fig. 3. MSE performance comparisons when  $K = 32$  and  $N/K = 0.5$ .

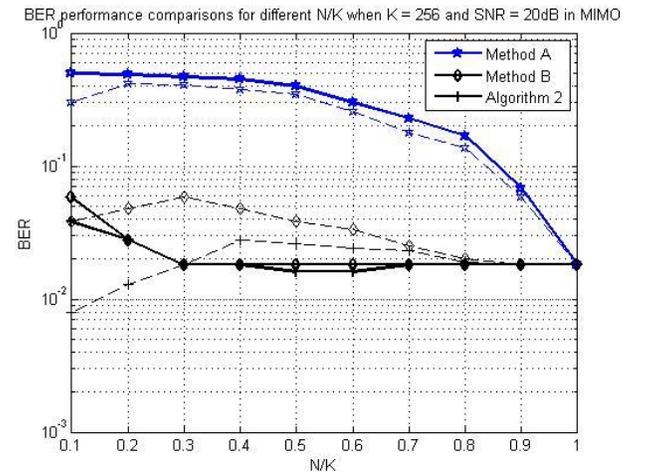


Fig. 4. BER performance comparisons when  $K = 32$  and  $N/K = 0.5$ .

The proposed Algorithms 1 and 2 have almost the same performance in both cases. The proposed algorithms outperform Method B, especially for Case 2. For example, compared with Method B, the proposed algorithms can provide about 10dB gain in SNR for MSE and about 8dB gain in SNR at a BER of  $10^{-2}$ . Moreover, compared with the optimal solution, the proposed algorithms only incur slight

performance degradation. We next consider a more practical OFDM-based CR system with  $K = 256$  subcarriers. The channel is assumed to have  $L = 16$  taps. The number of pilot tones is  $P = L = 16$  and the number of contiguous subcarriers in each sub-band for Case 2 is  $R = 8$ . The available spectral rate is still chosen as  $N/K = 0.5$ . For this scenario, the number of all possible pilot placements is  $(N_P) \approx 10^{20}$ . Exhaustively searching the desired one from all possible pilot placements is infeasible.

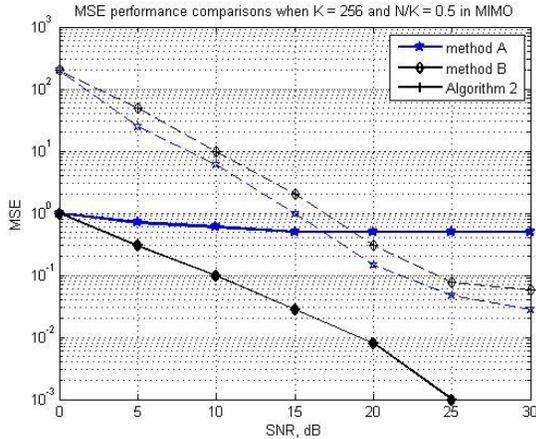


Fig.5.MSE performance comparisons when  $K = 256$  and  $N/K = 0.5$ .

Figures 5 and 6 show the MSE and BER performance of the three methods, respectively. It is seen that the proposed algorithm again outperforms the other two methods, especially in Case 2.

In CR systems, the available spectral rate  $N/K$  (i.e., the number of activated subcarriers  $N$ ) varies from time to time. Thus, we evaluate the three methods for different  $N/K$ . The BER performance comparisons are shown in Figure 7, where the SNR is set as 20dB. From the figure it can be seen that the proposed algorithm outperforms the other two methods, especially when  $N/K$  is small.

Here the channel frequency responses (CFR) at the pilot tones are first estimated and the CFR at the data subcarriers are then obtained by the frequency-domain interpolation. In our simulation, the spline interpolation is used. The results are shown in Figure 8 for  $N = 0.5K$ .

#### IV. CONCLUSION

In this paper, a new practical pilot design method for MIMO-based CR systems has been proposed. Using the Gerschgorin disk theorem, we have proposed a new formulation for the pilot placement problem with a simple objective function that involves only real additions. We have also proposed an efficient sequential method to solve the corresponding optimization problem. The performance of this pattern converges at almost 20dB SNR for different

filter lengths. Above this SNR value increasing the filter size has no influence any more on improving the performance. The longer the filter size is, the more virtual pilots are utilized in the filtering process, and this will result in accumulation of errors. Simulation results demonstrate that the proposed scheme outperforms two existing ones and more gains in SNR at a given BER can be obtained as the number of activated subcarriers decreases.

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